Geometer’s Sketchpad Tool to Promote Higher-Order Thinking Skills

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Abstract

There has been a call for the use of more higher-order thinking skills in the traditional mathematics classroom to help students succeed as opposed to the widely used memorization and recall methods (Marshall 2011). Bloom’s Taxonomy is a set of educational objectives used by educators to help students achieve these higher-order thinking skills (McBain 2011). The outdated traditional methods have not promoted the deep understanding of the material needed to progress on to the harder more in-depth mathematical concepts, often having left gaps in student understanding (Thompson 2017). In fact, researchers have found that students who are taught using computer-based instruction receive better test scores than those taught using the traditional lecture style (Nusir et al 2012).

Technology has become indispensable to the teaching and learning mathematics; it has enhanced lessons to improve student learning (Franz 2007). One such program is called Geometer’s Sketchpad. Geometer’s Sketchpad and similar information and communication technology programs have allowed students to quickly model, measure and calculate mathematical concepts. These tools have also allowed more time to explore the concepts in-depth, saving time over traditionally drawn and measured by hand (Nordin, et all 2010). Use of these programs have in turn allowed for more higher-order thinking skills to be addressed, thus having given students a deeper understanding of the mathematical concepts. Research has called for the need of technology in the classroom as a tool for inquiry, problem-solving and promotion of higher-order thinking skills (Awe 2007).
There are available materials and lessons to use with Geometer’s Sketchpad, but the question is whether the provided lessons have been addressing higher-order thinking skills. Specific lessons on congruence were chosen and rated against Bloom’s Revised taxonomy, Common Core State Standards, and a lesson design rubric. The results of this study found that most of the lessons were mainly focused on lower-level thinking skills.
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Chapter 1 – The Problem

The use of higher-order thinking skills has been generally lacking in the mathematics classroom with the tendency toward information recall as opposed to deeper understanding and problem solving (Sakshaug & Wohlhuter, 2010). In 2011, Marshall and Horton found students had a higher cognitive level resulting from exploring concepts of math and science and a lower cognitive level resulting from students being explained the concepts. Thus, calling for the need of inquiry-based instruction over traditional methods (Marshal & Horton 2011). The National Council of Teachers of Mathematics (NCTM) updated their principles and standards in 2000 to emphasize the importance of technology in the teaching and learning of mathematics, claiming through extensive research that has been done on the subject that the use of technology in the mathematics classroom impacts the mathematics taught and strengthens student learning (NCTM 2000). Although there is an abundance of technology tools available for teachers to use, there is a lack in teachers’ ability and training to utilize the tools available to them. This is especially true when pertaining to designing student lessons pertinent to the material to be addressed (Habre & Grundmeier 2007).

The technology tool utilized in this paper is Geometer’s Sketchpad (GSP). Geometer’s Sketchpad’s origins date back to the 1980’s to the Visual Geometry Project in which researchers were looking to create technology software to aid in teaching geometry. Key Curriculum Press published this first copy in 1993 (McGraw-Hill). Geometer’s Sketchpad has been a very innovative technology that can be used to promote a deeper understanding of concepts in the geometry classroom (Geometer’s Sketchpad, Reference Manual, 2001).
dynamic software has allowed students to utilize an approach to mathematics that is exploratory. Through the construction and manipulation of figures, the students have been able to test, analyze and form conjectures about the mathematics being studied. Since the use of Geometer’s Sketchpad saved time over traditionally drawing out geometric figures, then erasing and redrawing them, it has allowed more time for teaching and practicing higher-order thinking skills. (Nordin, et all 2010).

Benjamin Bloom created a taxonomy of educational goals. Higher-order thinking skills were based on the cognitive domain of this taxonomy; these skills included analysis, evaluation and creation (Burton 2010). Bloom’s Revised Taxonomy was a revision of his original objectives by one of his students, David Krathwohl (Wilson 2016). This revised taxonomy has been used as a tool for evaluating and revising educational materials to promote higher order thinking skills (Seaman 2011). The question was whether the available Geometer’s Sketchpad lessons were properly promoting higher-order thinking skills according to Bloom’s Revised Taxonomy.

**Purpose and Justification**

The purpose of this project was to evaluate whether the materials used in the geometry classroom promoted higher-order thinking skills. Research found that students became better problem solvers when using the higher-levels of Bloom’s Revised Taxonomy (Gambrell 1996). Problem solving in math develops skills outside the mathematics classroom, effecting how a student thinks, promoting persistence and curiosity as well as student confidence in approaching unfamiliar situations (NCTM 200). This project could be implemented to
improve best practices in the teaching of geometry. By having easy, ready-to-use materials that could promote higher-order thinking skills, this could open students’ minds for deeper understanding and further success in mathematics.

**Research Question**

Do the lessons that were aligned with the Common Core State Standards on congruence in geometry and created for Geometer's Sketchpad promote higher-order thinking according to Bloom’s Revised Taxonomy as interpreted for mathematics?

**Definition of terms**

**Bloom’s Revised Taxonomy (BRT):**

Constitutive Definition: Bloom’s Taxonomy is “a framework” that is used to classify the intended learning goals of the students for instructional lessons (Krathwohl 2002). In early 2000, David Krathwohl, and one of Bloom’s students, Lorin Anderson, initiated a revision of the original work; the revision was then published in 2001 (Wilson 2016). A chart interpreted for mathematics from Bloom’s Revised Taxonomy was used for this research (Bloom’s Revised 2016).

Operational Definition: In this project, “Bloom’s Revised Taxonomy” refers to the aforementioned publication, chart and its related research.
**Differentiated Instruction:**

Constitutive Definition: Definition as defined by Sparks 2015 is modifying lessons to tailor to students’ unique set of strengths, needs and interests (Sparks 2015).

Operational Definition: In this project, “differentiated instruction” refers to the use of technology such as Geometer’s Sketchpad to teach geometry as opposed to the traditional lecture style teaching of mathematics.

**Geometer’s Sketchpad:**

Constitutive Definition: The Geometer’s Sketchpad (GSP) is a computer software system for creating, exploring, and analyzing a wide range of mathematics concepts in the field of algebra, geometry, trigonometry, calculus, and other areas (Geometer’s Sketchpad, Reference Manual, 2001).

Operational Definition: In this project, Geometer’s Sketchpad (GSP) refers to Version 4.0 of the software.
Higher-order Thinking Skills (HOTS):

Constitutive Definition: No one definition exists for this term. For purposes of this project the following definition will be used: Higher-order thinking skills pertain to the combination of critical, logical, reflective, metacognitive, and creative thinking. These skills are triggered when students confront new problems, quandaries, or questions. Effective utilization of these skills produce valid conjectures, achievements, and resolutions that are a logical result of a students’ prior knowledge and experience, and encourages continued progress in these and separate intellectual skills (King, et. al. 1998).

Operational Definition: HOTS will be referenced according to Bloom’s Revised Taxonomy.

Mathematical Thinking:

Constitutive Definition: Mathematical thinking is often defined in terms of proofs and justifications or the thoughts and processes used to reach such proofs or a combination of the two (Lane 2011).

Operational Definition: In this project, “mathematical thinking” will refer to both the use of proof and justifications and processes used to reach them.
Chapter 2 – Review of Literature

Traditional Views of Mathematics

Traditional math has been taught in a lecture style format and has still been the norm today for which most mathematics has been taught. It generally started with the introduction of the concept to be learned followed by a series of practice problems. Students relied on their teachers and the text to provide examples, which they attempted to replicate and then again relied on their teacher or the text for confirmation that they have completed the math correctly. As a result of such lessons, students have been left with an outlook of mathematics as merely a string of random rules, given to them by teachers, who learned them from some unknown intellectual source (Van de Walle 2010). Memorization and recitation of facts have been the staples of the traditional math style. Students repeated problems with different numbers inserted into the equation over and over until they learned the process. If a student was failing to understand, then even more practice was assigned until the student succeeded. In this style, lessons rarely differed from one lesson to the next. It was more a matter of how many problems were practiced, the level of clarity in the teacher’s explanation of the material and the level of difficulty of the new concept learned. Math in this style was taught as a “collection of truths” to be instilled in the students through the repetition of designed practice problems (Kohn 1999).

Differentiating Instruction with the Use of Technology

Differentiated instruction has been a common term used in the educational sector for some time now. It has many definitions but the concept is universal. One definition as
defined by Sparks is adjusting lessons to fit to the particular needs, strengths and interests of each individual student (Sparks 2015). Differentiated instruction differed from traditional instruction in the sense that the learners are recognized as having individual learning needs as opposed to the one size fits all technique of the traditional teaching style. Technology has become an ever more increasing tool used for differentiating instruction in the classroom. There have been many studies on the effectiveness of using technology to improve mathematical understanding in the classroom. Although many studies tend to have had mixed results on the overall effectiveness, some results have shown some strategies to have been more effective than others. For example, in Roschelle’s summary of research studying the effectiveness of technology use in the teaching of math, he found, students’ learning improved through the use of technological programs that required the students to think critically about mathematics (Roschelle 2000). The author went on to compare this to the use of technology to practice repetitive skills in a more engaging manner by trying to make the math fun for the students which consequently seemed to decrease performance (Roschelle 2000). A study conducted in 2010, examined the use of computers and other technology to strengthen basic math skills. This study concluded a “noticeable increase in student mastery of basic mathematics” (Hudson, et. al. 2010). In general, both differentiation and technology have proved to be successful in increasing student achievement in the classroom. It was a matter of how specifically they have been implemented and to what purpose that determined the overall success.
The use of dynamic geometry software such as Geometer’s Sketchpad (GSP) became a popular tool for using technology to teach mathematical concepts. The Geometer’s Sketchpad (GSP) is a computer software system for creating, exploring, and analyzing a wide range of mathematics concepts in the field of algebra, geometry, trigonometry, calculus, and other areas (Geometer’s Sketchpad, Reference Manual, 2001). This software has allowed the user to model mathematical concepts through the construction and animation of visual representation of those concepts. As Teoh & Fong summarized in their research, using GSP has resulted in a positive increase in student performance for learning geometric concepts as opposed to students who solely used a textbook to learn the equivalent concepts (Teoh & Fong 2005). A study by Kesan & Caliskan provided similar positive results in favor of the effectiveness of having used GSP as a tool for improving mathematical understanding. Their study concluded that students who learned the content using GSP were more successful and had a better understanding of the content than those who were taught using only the traditional non-technology aided instruction (Kesan & Caliskan 2013).

Higher-Order Thinking Skills (HOTS)

In speaking on the matter of purpose in teaching mathematics, a common theme has been promoting understanding and success in all levels of mathematics education as the role of higher-order thinking skills. Higher-order thinking skills are critical thinking skills and though easily confused are different from problem solving skills. Problem solving skills as defined by Gonzalez, are skills used in mathematics that enable students to find solutions to
mathematical queries (Gonzalez 2012). The author went on to explain that these skills tend to be step-by-step procedures used to breakdown problems, many times word problems, into a set of steps needed to reach solution. These are a necessary part of mathematics instruction and are very useful in increasing mathematical success, but critical thinking is another skill set needed for students to learn and overcome new challenges (Gonzalez 2012). Crawford & Brown summarized in their paper an explanation that HOTS were actually a compilation of three different types of thinking skills: “content thinking, critical thinking, and creating thinking” (Crawford, Brown 2002). These skills were all based on a different level featured in Bloom’s Taxonomy. Krathwohl explained Bloom’s Taxonomy, as “a framework” for categorizing specific student learning goals as a result of the lessons used for instruction (Krathwohl 2002). This taxonomy created by Benjamin S. Bloom has been widely used and recognized as a tool throughout the educational community (Seasman 2011). Originally published in 1956, forty-five years later it was revised and now referred to as the revised taxonomy. Bloom’s Revised Taxonomy (BRT) was broken into six categories that each in turn have their own subcategories (Krathwohl 2002). BRT was used as a tool for evaluating HOTS in lesson plans using technology to teach and promote mathematical understanding.

Mathematical Thinking

HOTS are overarching skills used across all subject areas in education, but have been used specifically in conjunction with mathematical thinking. Mathematical thinking as with many other terms has different definitions amongst educators and mathematicians alike. Mathematical thinking has often been defined in terms of proofs and justifications or the
thoughts and processes used to reach such proofs or a combination of the two. Lane quotes
Polya’s 1954 definitions of demonstrative reasoning and plausible reasoning to categorize
and explain the two types of definitions of mathematical thinking. Demonstrative reasoning
focused on using mathematical facts and knowledge already accepted to be true and learning
through verifying this knowledge to be true. Plausible reasoning on the other hand was the
thinking that preceded demonstrative thinking in that it was the thinking used to create and
discover new mathematical knowledge (Lane 2011). In the classroom, the difference between
the two types of thinking would be a teacher presenting a fact and the students verifying that
fact to be truth versus a teacher who presented a problem and guided the students into
formulating their own conclusion, thus having discovered for themselves an already known
mathematical fact. For the purposes of this study mathematical thinking referred to the
combination of demonstrative and plausible thinking. As Mason pointed out in his summary
of Boaler’s studies, the environment in which mathematical thinking was learned was
important to the students’ understanding and use of mathematical thinking. Boaler’s study
concluded that those students who were subject to the use of exploratory and participatory
mathematical problem solving understood and used mathematical thinking even when not in
the classroom, while the students who received rehearsal based instruction did not (Mason
1997). Having shown that just as with the concepts of differentiation and technology,
successful teaching of mathematical thinking depended highly on the implementation of the
skill.
Bloom’s Revised Taxonomy and Mathematics

Thompson quoted in his research that even mathematics teachers who intended to teach and write higher-order thinking questions often wrote lower-order thinking questions instead (Thompson 2017). This may be the reason for the growing number of resource lists pertaining to Bloom’s Taxonomy as interpreted for mathematics. One such resource was used in this paper and the chart is included in the appendix. The resource states that a mathematics teacher found that having used the higher levels of Bloom’s Revised Taxonomy in pertaining to mathematics instruction, students demonstrated improved problem-solving skills. The chart in the appendix breaks down each of the six levels of Bloom’s Revised Taxonomy as interpreted to be used in the mathematics classroom (Bloom’s 2016).

EngageNY

EngageNY.org is a website developed and implemented in 2011 by the New York State Education Department. The site has an abundance of resources based around the Common Core State Standards, including curriculum modules, lesson plans, rubrics, etc. The website was designed to help both educator and parents to understand and implement the Common Core State Standards that have been adopted by many states across the country (NYSED 2013).
Chapter 3 – Methodology

After reviewing the literature in Chapter 2, the researcher developed a project design for evaluating Geometer’s Sketchpad lessons targeted towards teachers of high school geometry. The researcher chose three methods of analysis focusing on three different aspects of the lessons. First, the researcher planned to determine whether the lesson met the intended Common Core State Standards. Second, the lessons were to be evaluated according to what level of Bloom’s Revised Taxonomy the lesson used the most. Finally, the lesson was analyzed with respect to the overall quality for a mathematics lesson.

Description of the Project Design

The research was an evaluation of lesson plans using Geometer’s Sketchpad to teach a geometry unit on congruence that promoted higher-order thinking skills. The lesson plans were evaluated according to the following framework, and then analyzed to find if the intended criteria had successfully been satisfied. This project used Geometer’s Sketchpad 4.0 as the software edition for all the lesson plans. The framework for this project was broken down into three parts. One part is based on the provided rubric “Tri-State Quality Review Rubric for Lessons & Units: Mathematics – Version 4.2”, which was used to grade the individual lessons according to the guidelines of Geometry Course Overview for Congruence (G-CO) which was in turn based on the New York State Common Core Learning Standards for Mathematics and the PARCC Model Content Framework for Geometry as defined on the EngageNY website (NYSED 2013). The framework also included HOTS as outlined by Bloom’s Revised Taxonomy formulated for mathematics. The lessons were furthermore
assessed specifically according to whether the Common Core State Standards for congruence in geometry were met. An example revision of one of the lessons is provided as an additional framework for future research.

**Audience**

These lesson plans and materials were intended for the use of teaching high school level geometry to students with access to the Geometer’s Sketchpad program. The purpose of the materials was to provide teachers and students with easy to use lessons for learning geometric concepts beyond memorization, recall and recognition. Each lesson provided exploration and practice of the concept and supported learning and retention.

**Description of the analysis used**

The evaluated lesson plans were categorized, evaluated, and scored according to three separate variables: whether the standards were met as defined by Geometry Common Core State Standards; whether higher-order thinking skills were achieved as defined by Bloom’s Revised Taxonomy; and whether the lesson plan met quality standards as defined by the Tri-State Quality Review Rubric for Lessons & Units: Mathematics – Version 4.2. The scoring scales for each are defined below.

**Common Core State Standards Score**

The standards were assessed on a basic 3-point scale as follows: 0 – standard not met, 1 – standard partially met, 2 – standard met.
Bloom’s Revised Taxonomy Score

The higher-order thinking skills were assessed according to the most used level of Bloom’s Revised Taxonomy and were scored on a scale of 1-6 respective to the level. Bloom’s Revised Taxonomy levels were defined in further detail in the appendix, but were given the following levels: Level 1 – Remembering, Level 2 – Understanding, Level 3 – Applying, Level 4 – Analyzing, Level 5 – Evaluating, and Level 6 – Creating. Figure 1 shows the breakdown used for this analysis and can also be found in the appendix.

**Bloom’s Revised Taxonomy: Mathematics**

<table>
<thead>
<tr>
<th>New Terms</th>
<th>Actions</th>
<th>Learning Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Creating</strong>&lt;br&gt;(Putting together ideas or elements to develop an original idea or engage in creative thinking.)</td>
<td>Designing&lt;br&gt;Constructing&lt;br&gt;Planning&lt;br&gt;Producing&lt;br&gt;Inventing&lt;br&gt;Deviving&lt;br&gt;Making</td>
<td>Creating: (Generating new ideas, products, or ways of viewing things) How could we determine the number of pennies in a jar without counting them? Apply and integrate several different strategies to solve a mathematical problem. Design a new monetary system or an experiment for establishing ... Designing, constructing, planning, producing, inventing. Invent a machine to do a specific task. Develop a menu for a new healthy foods restaurant.</td>
</tr>
<tr>
<td><strong>Evaluating</strong>&lt;br&gt;(Judging the value of ideas, materials and methods by developing and applying standards and criteria.)</td>
<td>Checking&lt;br&gt;Hypothesizing&lt;br&gt;Critiquing&lt;br&gt;Experimenting&lt;br&gt;Judging&lt;br&gt;Testing&lt;br&gt;Debating&lt;br&gt;Monitoring</td>
<td>Evaluating: (Judging the value of a product for a given purpose, using definite criteria) Develop a proof ... and justify each step ... Using a definition ... determine ... Justifying a decision or course of action, checking, hypothesizing, critiquing, experimenting, judging What criteria would you use to evaluate if your answer is correct? Prepare a list of criteria to judge... Evaluate expressions.</td>
</tr>
<tr>
<td><strong>Analyzing</strong>&lt;br&gt;(Breaking information into its component elements to explore relationships.)</td>
<td>Comparing&lt;br&gt;Organizing&lt;br&gt;Deconstructing&lt;br&gt;Atributing&lt;br&gt;Questioning&lt;br&gt;Structuring&lt;br&gt;Integrating</td>
<td>Analyzing: (Breaking information into parts to explore understandings and relationships) Given a math word problem, determine the strategies that would be necessary to solve it. Write a paragraph describing the relationship ... How does ... compare to ... Comparing, organizing, deconstructing, interrogating, finding Design a survey to find out ... Graph your results. Use a Venn Diagram to show how two topics are the same and different. Translate between visual representations, sentences, and symbolic notation. Make predictions based on experimental or statistical data.</td>
</tr>
<tr>
<td><strong>Applying</strong>&lt;br&gt;(Using strategies, concepts, principles and ideas in new situations.)</td>
<td>Implementing&lt;br&gt;Carrying out&lt;br&gt;Using&lt;br&gt;Executing</td>
<td>Applying: (Using information in concrete situations) Compute the area of actual circles. Use the given graph to ... Choose and describe the best method to ... Using information in another familiar situation, implementing, carrying out, using, executing Draw a diagram which shows these fractions or take photographs of the fractions. Determine measures of central tendency and dispersion Write a journal entry. Write an explanation about this topic for others.</td>
</tr>
<tr>
<td><strong>Understanding</strong>&lt;br&gt;(Understanding of given information.)</td>
<td>Interpreting&lt;br&gt;Exemplifying&lt;br&gt;Summarizing&lt;br&gt;Inferencing&lt;br&gt;Paraphrasing&lt;br&gt;Classifying&lt;br&gt;Comparing&lt;br&gt;Explaining</td>
<td>Understanding: (Grasping the meaning of material) Given the mathematical formula for the area of a circle, paraphrase it using your own words. Select the graph that illustrates Explaining ideas or concepts Interpreting, summarizing, paraphrasing, classifying, explaining Find items that you can use to show the fractions. Relate or write in your own words ... Report to the class... Write a summary report of the event.</td>
</tr>
<tr>
<td><strong>Remembering</strong>&lt;br&gt;(Recall or recognition of specific information.)</td>
<td>Recognizing&lt;br&gt;Listing&lt;br&gt;Describing&lt;br&gt;Identifying&lt;br&gt;Retrieving&lt;br&gt;Naming&lt;br&gt;Locating&lt;br&gt;Finding</td>
<td>Remembering: (Remembering previously learned material) State the formula for the area of a circle. State the rules of ... Explain and use the procedure for ... Recalling information, recognizing, listing, describing, retrieving, naming, finding, locating List the fractions you know and can show. List the attributes of your shape. Make a concept map of the topic Make a chart showing...</td>
</tr>
</tbody>
</table>

*Figure 1*
Rubric Score

The quality of each lesson was assessed according to the rubric: “Tri-State Quality Review Rubric for Lessons & Units: Mathematics – Version 4.2” designed by the Tri-State Collaborative. Figures 2 & 3 display the rubric and are also included in the appendix. This rubric was broken into four different dimensions: Alignment to the Rigor of the CCSS, Key Areas of Focus in the CCSS, Instructional Supports, and Assessment. Each dimension had a list of criteria, including “must have” criteria that were marked with a double asterisk (**). These “must have” criteria were the basis of the rating system. Each dimension was rated 0-3, according to the following scale: 3: Meets all “must have” criteria (**) and most of the other criteria in the dimension; 2: Meets many of the “must have” criteria and many of the other criteria in the dimension; 1: Meets some of the criteria in the dimension; 0: Does not meet the criteria in the dimension. The three separate scores were evaluated and the lesson was given an overall rating according to the scale defined on the rubric: E: Exemplar Lesson – meets all the “must have” criteria and most of the other criteria in all four dimensions (mainly 3’s); E/I: Exemplar if Improved – needs some improvement in one or more dimensions (mainly 3’s and 2’s); R: Needs Revision – is a “work in progress” and requires significant revision in one or more dimensions (mainly 2’s and 1’s); N: Not recommended – does not meet the criteria in the dimensions (mainly 1’s and 0’s).
GEOMETER’S SKETCHPAD TOOL TO PROMOTE HIGHER-ORDER THINKING

Tri-State Quality Review Rubric for Lessons & Units: Mathematics – Version 4.2

<table>
<thead>
<tr>
<th>Grade: Mathematics Lessons/Unit:</th>
<th>Overall Rating:</th>
</tr>
</thead>
<tbody>
<tr>
<td>III. Instructional Supports</td>
<td></td>
</tr>
<tr>
<td>Observations and Comments:</td>
<td></td>
</tr>
<tr>
<td>- The lesson/unit meets the criteria for all four dimensions.</td>
<td></td>
</tr>
<tr>
<td>- The lesson/unit demonstrates the &quot;must have&quot; criteria and most of the other criteria in all four dimensions.</td>
<td></td>
</tr>
<tr>
<td>- The lesson/unit is a &quot;work in progress&quot; and requires continued development and refinement.</td>
<td></td>
</tr>
<tr>
<td>- The lesson/unit does not meet the criteria in any dimension (minor 1 and 2).</td>
<td></td>
</tr>
<tr>
<td>IV. Assessment</td>
<td></td>
</tr>
<tr>
<td>Observations and Comments:</td>
<td></td>
</tr>
<tr>
<td>- The lesson/unit requires assessment and feedback at all levels.</td>
<td></td>
</tr>
<tr>
<td>- The lesson/unit is a &quot;work in progress&quot; and requires continued development and refinement.</td>
<td></td>
</tr>
<tr>
<td>- The lesson/unit does not meet the criteria in any dimension (minor 1 and 2).</td>
<td></td>
</tr>
</tbody>
</table>

Rating: 5 4 3 2 1

Overall Rating: 5 4 3 2 1

Figure 2

Tri-State Quality Review Rubric for Lessons & Units: Mathematics – Version 4.2

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</tr>
</tbody>
</table>

Rating: 5 4 3 2 1

Overall Rating: 5 4 3 2 1

Figure 3
This type of research was intended to set guidelines for evaluating and improving lessons to include the use of higher-order thinking skills. The research is intended to be used in teaching high school geometry, specifically pertaining to the use of technology such as Geometer’s Sketchpad and similar programs. The results of the analysis of the selected lessons are outlined in the following chapter.
Chapter 4 – Analysis

The question to be answered is whether pre-made lessons pertaining to congruence standards of the Common Core State Standards properly promote higher-order thinking skills as outlined in accordance to Bloom’s Revised Taxonomy and interpreted for mathematics.

Each lesson is scored according to three mentioned categories: Common Core State Standards (CCSS Score), Bloom’s Revised Taxonomy, and Rubric Score. The Common Core Standards are scored as follows and uses a 3-point scale: 0 – standard not met, 1 – standard partially met, 2 – standard met.

**Bloom’s Revised Taxonomy: Mathematics**

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<th>Higher-order thinking</th>
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<tr>
<td>(Putting together ideas or elements to develop an original idea or engage in creative thinking)</td>
<td>Creating: (Generating new ideas, products, or ways of viewing things) How could we determine the number of vertices in a jar without counting them? Apply and integrate several different strategies to solve a mathematical problem. Design a new monetary system or an experiment for establishing ... Designing, constructing, planning, producing, inventing. Invent a task to do a specific task. Develop a menu for a new healthy foods restaurant.</td>
</tr>
<tr>
<td>Evaluating</td>
<td>Checking, Hypothesizing, Critiquing, Experimenting, Judging, Testing, Detecting, Monitoring</td>
</tr>
<tr>
<td>(Judging the value of ideas, materials and methods by developing and applying standards and criteria)</td>
<td>Evaluating: (Judging the value of a product for a given purpose, using definite criteria) Develop a proof ... and justify each step ... Using a definition ... determine ... Justifying a decision or course of action, checking, hypothesizing, critiquing, experimenting, judging. What criteria would you use to evaluate if your answer is correct? Prepare a list of criteria to judge ... Evaluate expressions.</td>
</tr>
<tr>
<td>Analyzing</td>
<td>Comparing, Organizing, Deconstructing, Attributing, Outlining, Structuring, Integrating</td>
</tr>
<tr>
<td>(Breaking information into its component elements to explore relationships)</td>
<td>Analyzing: (Breaking information into parts to explore understandings and relationships) Given a word problem, determine the strategies that would be necessary to solve it. Write a paragraph describing the relationship ... How does ... compare to ... Comparing, organizing, deconstructing, interrogating, finding. Design a survey to find out ... Graph your results. Use a Venn Diagram to show how two topics are the same and different. Translate between visual representations, sentences, and symbolic notation. Make predictions based on experimental or statistical data.</td>
</tr>
<tr>
<td>Applying</td>
<td>Implementing, Carrying out, Using, Executing</td>
</tr>
<tr>
<td>(Using strategies, concepts, principles and theories in new situations)</td>
<td>Applying: (Using information in concrete situations) Compute the area of actual circles. Use the given graph to ... Choose and describe the best method to ... Using information in another familiar situation, implementing, carrying out, using, executing. Draw a diagram which shows these fractions or take photographs of the fractions. Determine measures of central tendency and dispersion. Write a journal entry. Write an explanation about this topic for others.</td>
</tr>
<tr>
<td>Understanding</td>
<td>Interpreting, Summarizing, Inferring, Paraphrasing, Classifying, Comparing, Explaining</td>
</tr>
<tr>
<td>(Understanding of given information)</td>
<td>Understanding: (Grasping the meaning of material) Given the mathematical formulas for the area of a circle, paraphrase it in your own words. Select the graph that illustrates explaining ideas or concepts. Interpreting, summarizing, paraphrasing, classifying, explaining. Find items that you can use to show the fractions. Recall or write in your own words ... Report to the class ... Write a summary report of the event.</td>
</tr>
<tr>
<td>Remembering</td>
<td>Recognizing, Listing, Describing, Identifying, Revising, Naming, Locating, Finding</td>
</tr>
<tr>
<td>(Recall or recognition of specific information)</td>
<td>Remembering: (Remembering previously learned material) State the formula for the area of a circle. State the rule of ... Explain and use the procedure for ... Recalling information, recognizing, listing, describing, retrieving, naming, finding, locating. List the fractions you know and can show. List the attributes of your shape. Make a concept map of the topic. Make a chart showing ...</td>
</tr>
</tbody>
</table>
Next, Bloom’s Revised Taxonomy is scored according to which level of the taxonomy was most used in the lesson. Bloom’s Revised Taxonomy has six levels and the rubric used was framed specifically to mathematics. This rubric can be seen in Figure 4 and again in the appendix.

Additionally, the overall quality of the lessons was evaluated according the Tri-State Quality Rubric in Figures 5 & 6 and is also included in the appendix. This rubric was broken down into four dimensions to examine that the overall lessons included alignment and focus of CCSS, support for instruction and appropriate assessment. The Results of the scores for each lesson are provided in a table at the end of the individual lesson analysis. Finally, an Example Revision is provided for one of the lessons as a guide for future revision of lessons.

![Figure 5](image-url)
Lesson Analysis

Altitudes in a Triangle

CCSS Score

This lesson addressed the following CCSS for Geometry:

**CCSS.Math.Content.HSG.CO.C.10** - Prove theorems about triangles.

**CCSS.Math.Content.HSG.CO.D.12** - Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
This lesson successfully met both the stated standards through construction and analysis. The students explored and analyzed the properties of triangles relating specifically to their altitudes. Thus, this lesson received a score of 2.

**Bloom’s Revised Taxonomy Score**

This sketchpad started out asking the student to recognize different types of triangles and moved into understanding the relationships of altitudes in different types of triangles. Progressing through the worksheet, it asks the student to analyze by asking the student what he noticed about the relationships of the altitudes in the different types of triangles. The final question asked the student to create different area formulas from what he learned. This reached the highest-order of thinking according to Bloom’s Revised Taxonomy. As a whole, the lesson covered levels 1, 2, 4 and 6 in BRT, but focused mostly on analysis. Thus, this lesson scored a level 4.

**Rubric Score**

For Alignment to the Rigor of the CCSS, it rated as a 3, having met all criteria in this section. For Key Areas of Focus in the CCSS, it covered Focus, but as far as Coherence, it failed to make connections across domains. As far as Application was concerned, the lesson made no reference to real-world situations. The lesson, however, appropriately addressed Conceptual Understanding and Procedural Skill and Fluency. Since the “must have” and most other criteria were met, this dimension also received a score of 3. For Instructional Supports, this lesson met both the “must haves”. Although the lesson was easy to use and elicited mathematical thinking, it was lacking in stimulating interest, providing extra support for
students working below grade level, and could have used more extensions for those working above grade level. The scoring for this dimension was a 2, since it lacked support criteria pertaining to interest and material support for underachieving and overachieving students. For Assessment, although the lesson used grade appropriate language and allowed the student to demonstrate the targeted CCSS, it lacked in clear grading. It provided an answer key but did not provide a rubric or guidance for interpreting student performance. Thus, it received a score of 1 for only meeting a small proportion of criteria for this dimension. This overall lesson was scored E/I: Exemplar if Improved. It overall met and properly addressed the CCSS, but it was lacking somewhat in the Instructional Supports and greatly in the Assessment dimensions.

**Constructing Regular Polygons**

**CCSS Score**

This lesson addressed the following CCSS for Geometry:

*CCSS.Math.Content.HSG.CO.A.2* - Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs.

*CCSS.Math.Content.HSG.CO.A.5* - Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software.

*CCSS.Math.Content.HSG.CO.D.13* - Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
This lesson was rated as a 1 in this category for the given CCSS for geometry. Since the lesson focused mainly on constructions through rotations and left out other types of transformations, such as reflections and translations. Although it used the concept of the circle to create regular polygons, it did not directly link the concept of inscribed circles.

**Bloom’s Revised Taxonomy**

This lesson focused mainly on the application level of BRT. The lesson had the student apply his knowledge of rotations and central angles to create various regular polygons. Although it used some level of creating in having had the student derive an expression for a polygon with \( n \) number of sides, most of the lesson still focused only on application. Thus, this lesson scored a 3.

**Rubric Score**

For Alignment to the Rigor of the CCSS, the score was a 1. Since as mentioned previously, the lesson did not fully address the listed CCSS for geometry, it only met some of the criteria for this dimension. For Key Areas of Focus in the CCSS, it did not meet the “must have” of this section. Although it did center on all the concepts of the listed CCSS, it did not provide an appropriate level of rigor. In addition, it did not allow students to transfer skills across domains or provide real-world connections. The problem solving required was not complex or challenging enough, having focused more on lower-order thinking skills. Thus, the lesson received a 1 for this dimension as well. For Instructional Supports, the lesson provided a fair amount of guidance and used appropriate and useful representations. However, this lesson lacked in interest and further support. For example, it did not provide adequate support for
either under-achieving or over-achieving students. This dimension received an overall rating of 2. For Assessment, an answer key was provided but beyond that there was little assessment of proper use of the targeted CCSS and overall no guidelines for assessment, thus having received a score of 1. Receiving mostly 1’s, this lesson overall received a score of R: Needs Revision.

**Exterior Angles in a Polygon**

**CCSS Score**

The following lesson addressed the following CCSS for Geometry:

*CCSS.Math.Content.HSG.CO.C.9* - Prove theorems about lines and angles.

*CCSS.Math.Content.HSG.CO.D.12* - Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

This lesson received a 2 for the score in this section since it covered both the intended standards.

**Bloom’s Revised Taxonomy Score**

This was a short lesson in terms of the amount of direction provided, but since it had the student writing his own conjecture, explaining and investigating, it received high scores for requiring the student to use higher-order thinking skills. Since the lesson used skills from both levels 5 and 6 of Bloom’s Revised Taxonomy, it received a score of 6 for focusing more at that level.
Rubric Score

For Alignment to the Rigor of the CCSS, the lesson received a 3, having met all the criteria for this dimension. For Key Areas of Focus in the CCSS, it met the “must have” but failed to make connections across domains and connections to real-world situations. It was very strong in Conceptual Understanding. It received a 2 for this dimension. For Instructional Support, this lesson lacked in provoking interest. It showed no support for cultural and linguistic background or for students performing below grade level. Furthermore, it did not address any extensions for students performing above grade level. However, it did meet the “must haves” of the dimension in its clear guidance, ease of use, use of technology and meeting the targeted standards. Thus, the score for this dimension was a 2. For Assessment, although it was well designed to show the student demonstrates the targeted CCSS, it again did not provide a rubric or scoring guidelines past an answer key. Thus, it scored a 2 for this section. Overall this lesson scored an E/I: Exemplar if Improved, having received mostly 2’s and a 3.

Introducing Transformations

CCSS Score

This lesson addressed the following CCSS for Geometry:

- **CCSS.Math.Content.HSG.CO.A.4** - Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

- **CCSS.Math.Content.HSG.CO.A.5** - Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software.
This lesson had students explore all three types of transformations. Through comparisons it helped the students understand the definitions and properties of each of the three types of transformations. Using geometry software, the students created and transformed figures. Since the lesson meets all the stated standards above, it received a 2.

**Bloom’s Revised Taxonomy Score**

This lesson focuses on the level 2 – Understanding tier of Bloom’s Revised Taxonomy, since the main focus was on comparing and summarizing the three types of transformations. There was some application involved towards the end of the lesson, but since the main focus was all based around understanding, this lesson received a score of 2.

**Rubric Score**

For Alignment to the Rigor of the CCSS dimension, this lesson received a 3. It met all the “must have” criteria and covered all the stated standards for the lesson. For Key Areas of Focus in the CCSS, this lesson did a good job of having students demonstrate conceptual understanding, but lacked in providing genuine real-world connections or connections across domains. Since it met all the “must haves” and most other criteria, it received a 3 for this dimension. For Instructional Supports, this lesson met the “must have” criteria. It provided clear guidance through the lesson and used technology to explore the terminology and graphical representations of the three different types of transformations. However, it did not differentiate learning for those who may struggle or include extensions for those beyond grade level. Aside from the use of technology it had no aspects to spark interest or connect to diverse cultural backgrounds. This dimension received a 2. For Assessment, it allowed the
student to demonstrate the targeted standards and provided an answer key but no rubrics to assess student proficiency on the topic. For this dimension, a 2 was assigned. This overall lesson was graded as E/I: Exemplar if Improved, having received 3’s and 2’s.

**Proportions with an Angle Bisector in a Triangle**

**CCSS Score**

This lesson addressed the following CCSS for Geometry:

*CCSS.Math.Content.HSG.CO.C.10* - Prove theorems about triangles.

*CCSS.Math.Content.HSG.CO.D.12* - Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

This lesson explored theorems about proportions with angle bisectors in triangles. It had the student write a conjecture about their findings, using a process that formulated a theorem. Therefore, it met the requirements of the stated standards and received a 2.

**Bloom’s Revised Taxonomy Score**

This lesson focused mainly on understanding and applying knowledge of the given concept. It ended in the student writing a conjecture, which is a high-order thinking skill. Although it did include higher-order thinking, most of the questions were based on lower-order thinking skills of level 2, so this lesson received a 2 in this category.

**Rubric Score**

For Alignment to the Rigor of the CCSS, this lesson received a 3. It met all the “must have” criteria and walked the student step-by-step through formulating a theorem. It used graphical
representation to form a conceptual understanding of the intended subject matter. For Key Areas of Focus in CCSS, the lesson provided a good focus and supported the student in learning the mathematical concepts. It did not provide any real-world connections or related problem solving. Since it covered the “must haves” and most other criteria, it received a 3 for this dimension. For Instructional Supports, this lesson used technology to support and guide the student through learning the content stated in the standards. This lesson lacked in reaching a broad range of learners and in providing an aspect of interest to get the students motivated in the learning process. However, since it did meet the “must have” criteria and some of the other criteria it received a 2 for this dimension. For Assessment, this lesson guided the student through to proficiency of the targeted standards and included an answer key but no rubrics for interpreting student proficiency. Therefore, this lesson received a 2 in this dimension. This lesson received an overall score of E/I: Exemplar if Improved, since it received mostly 3’s in the stated dimensions.

**Triangle Congruence**

**CCSS Score**

This lesson addressed the following CCSS for Geometry:

*CCSS.Math.Content.HSG.CO.B.7* - Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

*CCSS.Math.Content.HSG.CO.B.8* - Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
This lesson explored the definition of congruence when pertaining to triangles. It had the students determine the criteria of angle and side patterns that produce congruent triangles. Since it met the above stated standards, it received a 2 for this category.

**Bloom’s Revised Taxonomy Score**

This lesson focused on higher-order thinking skills. Most the questions revolved around the analysis tier, having had the students make inferences and support their conjectures with explanations. There was some understanding and application; however, since most of the lesson focused on analysis the lesson received a 4 for this category.

**Rubric Score**

For Alignment to the Rigor of the CCSS, this lesson met all the “must have” criteria. It presented the students with a basis for a deeper understanding of the concepts of triangle congruence and received a score of 3 for this dimension. For Key Areas of Focus in CCSS, this lesson thoroughly centered on the stated standards. It promoted conceptual understanding and application. Although it lacked in modeling real-world situations and addressing skills across domains, it met many of the criteria of this dimension and received a score of 2. For Instructional Supports, this lesson gave guidance to the students helping them achieve understanding of the standards through the support of technology. It lacked in differentiation and stimulating interest. Since it met many the criteria of this section it received a 2 for this dimension. For Assessment, this lesson provided an answer key and guidance that led students to correctly identify which patterns would always create congruent triangles. The lesson was assessable and used appropriate prompts for the grade level. Since it met the
criteria of this section it received a 3. The overall lesson having received a mix of 2’s and 3’s in all dimensions received an E/I: Exemplar if Improved.

**Triangle Inequalities**

**CCSS Score**

This lesson addressed the following CCSS:

*CCSS.Math.Content.HSG.CO.C.10* - Prove theorems about triangles.

This lesson had the student exploring a theorem pertaining to the side lengths and angle measures of triangles. This met the stated standards and received a 2 for this category.

**Bloom’s Revised Taxonomy Score**

This lesson focused only on lower-order thinking skills. Although it did have some tier 3 – Application involved, the main focus of the lesson was centered in the tier 2 – Understanding of Bloom’s Revised Taxonomy. Therefore, the lesson received a 2 for its score in this category.

**Rubric Score**

For Alignment to the Rigor of the CCSS, this lesson met its targeted standard. It explored a specific theorem, stating that the sum of the measures of two sides of a triangle was greater than the third side. It received a 3 for this section. For Key Areas of Focus in the CCSS, the lesson included proper focus on the standard. It also promoted fluency of the skill and conceptual understanding; however, it lacked in rigor and persistence in problem solving for the intended grade level expectation. Thus, this lesson received a 2 for this dimension. For Instructional Support, this lesson could be considered too easy for the intended grade level. It
lacked any differentiation especially for those performing above grade level. It did not stimulate interest or thought-provoking questions. Since it met the “must have” criteria, but only some of the other criteria, it received a 2 for this dimension. For Assessment, this lesson came with an answer key and gave observable evidence that the intended skill set was achieved. It did not include a rubric for scoring the level of proficiency and understanding achieved. Since it met most of the stated criteria it received a 2 for this section. The overall lesson received an E/I: Exemplar if Improved.

Results

The results of the scores of the individual lessons were recorded in the following table.

<table>
<thead>
<tr>
<th>Lesson</th>
<th>CCSS Score</th>
<th>BRT Score</th>
<th>Rubric Score</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitudes in a Triangle</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Constructing Regular Poly.</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Exterior Angles in a Polygon</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Introducing Transformations</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Proportions with an Angle Bis.</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Triangle Congruence</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Triangle Inequalities</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

The results showed that less than half of the lessons had a main focus on higher-order thinking. Many lessons only focused on level 2 – Evaluating of Bloom’s Revised Taxonomy.
Although some of those lessons did have at least one question in the higher levels of the taxonomy, most of the questions were focused on lower levels of thinking. Only one lesson focused the highest level of Bloom’s Revised Taxonomy. Most of the lessons, however, met the Common Core State Standards and were well rounded. There was only one lesson that fell short of meeting its intended Common Core State Standard. According to the standards of the Tri-State Quality Review Rubric, most of the lessons still had room for improvement. One lesson in particular scored very poorly according to this rubric.
Example Revision

The following was a revision of the lesson entitled, “Constructing Regular Polygons”.

This lesson was chosen due to its low scores in the three categories of evaluation. The following revision was designed to provide an example for future revisions.

Revision of “Constructing Regular Polygons”

Introduction

Is Geometry Everywhere?

Have a PowerPoint with slides of examples of polygons in the real-world. Ask the students to write two lists: one of objects that have that contain regular polygons; the second of professions they think would use regular polygons. (Optional: Allow them to browse the internet). Have them share their list with a partner or small group. Allow students to share with the class the responses which are the most unique/creative responses.

Lesson

Use the provided lesson “Construction Regular Polygons” with the addition of the following problems:

- Prove your conjecture for the interior angle measure of a regular $n$-gon to always be true.

- Using your knowledge of compass and straight edge constructions, plan a test to find which regular polygons can be constructed using only a compass and straight edge. Perform your test on two regular polygons; present and justify your results.
- Create a method for inscribing regular polygons in a circle. Theorize if all regular polygons can be inscribed in a circle by one method. Test your theory and justify the results. Construct a regular triangle, square and hexagon inscribed in a circle.

- Predict which regular polygons can form regular tessellations. Create the tessellations using regular polygons. Justify why the ones you created are the only possible regular polygons to form regular tessellations. Determine which of the three transformations or combinations of them can be used to create the tessellation.

- Predict which combination of regular polygons can form semi-regular tessellations. Create a tessellation for each combination. Justify why the ones you created are the only possible combinations. Determine which of the three transformations or combinations of them were used to create each tessellation. All three must be used at least once.

- Choose three real-world situations in which regular polygons are used that interest you. Then research it and present your findings to the class. Theorize why regular polygons are used in each situation as opposed to other designs.

**Design Project**

Design a stained-glass window using a semi-regular tessellation using at least three different colors. Your design must fit in a 24 x 48 in window. Calculate the amount of metal needed to finish the design as well as the area of the glass needed for each color.
Assessment

The following rubric can be used to assess the students’ performance:

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Completion</strong></td>
<td>The student thoroughly completed the problem.</td>
<td>The student completed most of the problem.</td>
<td>The student completed some of the problem.</td>
<td>The student did not complete any part of the problem.</td>
</tr>
<tr>
<td><strong>Accuracy</strong></td>
<td>The student has completely correct solutions/statements.</td>
<td>The student has some incorrect solutions/statements.</td>
<td>The student has many incorrect solutions/statements.</td>
<td>The student has no correct solutions/statements.</td>
</tr>
<tr>
<td><strong>Strategy</strong></td>
<td>The student’s work is logical and leads to the correct solution.</td>
<td>The student’s work has some flaws but leads to the correct solution.</td>
<td>The student’s work has flaws and leads to an incorrect solution.</td>
<td>The student’s work does not relate to the task.</td>
</tr>
<tr>
<td><strong>Delivery</strong></td>
<td>The student explained the reasoning used to reach the solution and it is clear and makes sense.</td>
<td>The student explained the reasoning used to reach the solution and it is somewhat clear.</td>
<td>The student explained the reasoning used to reach the solution and it is not clear.</td>
<td>The student did not explain the reasoning used to reach the solution.</td>
</tr>
<tr>
<td><strong>Optional: Creativity</strong></td>
<td>The student’s solution is unique and imaginative and appropriate to the problem.</td>
<td>The student created a new idea/design appropriate to the problem.</td>
<td>The student successfully adapted an example.</td>
<td>The student successfully reproduced an example.</td>
</tr>
</tbody>
</table>

**Support:**

For those performing below grade-level support can be provided through the following examples:

- working with a partner or in small groups
- pre-made tessellations could be provided to examine and evaluate
• manipulatives of regular polygons can be given as support

• provide hints, guidance, or review of previous content necessary to complete assignment

For those performing above grade-level examples of further challenges are provided below:

• have students try to create as many regular polygons as possible using only compass and straight edge and explain why some are impossible

• have students extend knowledge to two three-dimensional shapes and platonic solids

• have students extend knowledge to non-Euclidean geometries

• provide challenge problems using the learned knowledge

  ex. If three regular polygons (pentagon, hexagon and octagon) are inscribed in a circle with radius of 1, the regular polygons meet so that one side from each forms a triangle. Find the area of the triangle.

**Notes on the Revision**

The lesson was revised to improve the weaknesses that were addressed in the original analysis of the lesson. The following changes were added by category/dimension:

**CCSS**

Problems were added that included the missing transformations through the use of tessellations. The student is now required to use each of the three types of transformations as well as to evaluate which ones can be used in different tessellations. Furthermore, the student
is required to create regular polygons inscribed in circles and evaluate and establish a method for doing so.

**Bloom’s Revised Taxonomy**

Since the original lesson focused mainly on tier 3 – Application, problems were added so that the main focus is shifted towards tier 5 – Evaluating and tier 6 – Creating. The students now have many problems that require them to: design, create and construct; theorize, plan and test; as well as prove, defend and justify.

**Rubric**

For Alignment to the Rigor of the CCSS, the lesson was modified to meet all the criteria for the stated Common Core State Standards. For Key Areas of Focus in the CCSS, not only was the rigor increased, adding more of a challenge, but the lessons were modified to include real-world connections to the material. For Instructional Supports, some interest was added in the additional problems, giving the students some choices and requiring more creative work beyond application. Additionally, ideas and suggestions for under-achieving and over-achieving students were provided. For Assessment, to improve upon the provided answer key, a rubric was designed to evaluate student solutions and responses.

**Conclusion**

Seven lessons were chosen and evaluated according to three categories. First, the lessons were analyzed on whether they met Common Core State Standards for congruence. Most lessons did well, leaving only one that did not meet all its standards. Second, the lessons were graded according to the level of Bloom’s Revised Taxonomy as classified
according to mathematics. Many of the lessons did poorly, only focusing on the lower levels of thinking. Finally, the lessons were inspected according to the four dimensions of the Tri-State Quality Review rubric. Only one of the lessons was very weak but all of them needed some sort of improvement in the end. Furthermore, one of the lessons was chosen as an example and revisions were made to meet the weaknesses found in the analysis. Further discussion of the results can be found in Chapter 5.
Chapter 5 – Discussions and Conclusions

This section discusses the results of the analysis of seven chosen Geometer’s Sketchpad lessons. These Geometer’s Sketchpad lessons were evaluated to determine whether they were aligned with the Common Core State Standards on congruence in geometry and if the lessons promote higher-order thinking according to Bloom’s Revised Taxonomy as interpreted for mathematics. The research was separated into three methods for evaluation and scored according to the rubrics defined in Chapter 3. These three scores were the following: Common Core State Standards Score, Bloom’s Revised Taxonomy Score and Tri-State Quality Review Rubric Score.

When considering the results of this study, Awe’s research on the use of Geometer’s Sketchpad to aid in student learning should be kept in mind. Awe found that using Geometer’s Sketchpad improves learning for all students. He discovered that pairs of students who used Geometer’s Sketchpad performed better than those using only traditional methods (Awe 2007). Additionally, Bakry’s research found that the students with higher ability in math are able use the higher-order levels of Bloom’s taxonomy, such as creating meaning and forming opinions (Bakry 2015). Thus, the use of Geometer’s Sketchpad and higher-order thinking skills have both been proven to improve students’ mathematical competence. This supports that use of Geometer’s Sketchpad to improve higher-order thinking in the geometry classroom should result in increased mathematical performance. Thus, revising lessons in Geometer’s Sketchpad to promote higher-order thinking could produce valuable resources for teachers of geometry.
Of the seven evaluated lessons all pertaining to Congruence standards of the Common Core State Standards for geometry, only one lesson fell short of meeting all aspects of the associated standards for that lesson.

On the other hand, the Bloom’s Revised Taxonomy scores were varied across the board. Unfortunately, less than half of them ended with scores in the higher-order thinking tiers of 4 – Analyzing, 5 – Evaluating, and 6 – Creating. Whereas most of the lessons had at least one or more higher-order thinking questions in the lesson, the majority of the questions were otherwise focused on lower-order thinking skills of 1- Remembering, 2 – Understanding, and 3 – Applying. The most common emphasis was based around the Understanding tier. One lesson rose above the others as truly focusing on the highest tier of higher-order thinking skills. That was the Exterior Angles in a Polygon lesson, having received the highest possible score of six for having the students doing the most creative work.

In pertaining to the rubric scores, according to the Tri-State Quality Review Rubric, the lessons all scored very similarly, except for one lesson that scored quite poorly. The Constructing Regular Polygons failed to meet most of the criteria set out in each of the four dimensions. All the other lessons scored similarly and were lacking in the same key aspects. Aside from the interest sparked purely by being able to use technology to explore mathematical concepts, none of the lessons were relatable beyond mathematics and most lacked connections between different mathematical concepts as well. Furthermore, the use of problem solving skills and relationships to real-world situations were non-existent. Although some lessons required more complex problem solving than others, most guided the student
step-by-step through to the intended conclusions. This was doing too much of the problem-solving aspects for them. This was why they were rated low on Bloom’s Revised Taxonomy many times. In addition, all the lessons also suffer from differentiation, not providing support for students performing below grade level and not stimulating enough of a challenge for students who are performing above grade level. As far as assessment is concerned, even though all the lessons provided answer keys, there are different levels of complexity to the answers that are not accounted for. An improvement would be to provide a rubric rather than a right or wrong answer, especially when the students are asked to write conjectures, explain or defend their answers.

Overall these lessons rated well according to whether they met the Common Core State Standards and as evaluated by the Tri-State Quality Review Rubric for Lessons. According to Bloom’s Revised Taxonomy, the lessons scored poorly in consistently reaching a focus on the upper tiers of higher-order thinking. Future action can be directed towards the revision of these lessons. This endeavor should have a definitive focus on emphasizing higher-order thinking skills and correcting the weakness according to the Tri-State Quality Review Rubric.

Once the weaknesses are addressed and the focus of the lessons are shifted to focus on higher-ordering thinking, these lessons and similar lessons revised to these standards should be researched in practice in the classroom. Further research should be conducted with student comprehension and test results comparing the use of Geometer’s Sketchpad lessons that focus on lower-order thinking skills with these revised lessons that focus on higher-order thinking skills. These results may also be compared with traditional teaching styles to show
reproducible evidence that these lessons which use a combination of Geometer’s Sketchpad and higher-order thinking skills help improve student learning and comprehension of mathematical concepts.
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Appendix: Project Resources
Common Core State Standards (CCSS Materials)

Geometry Overview

The content standards associated with Geometry are based on the New York State Common Core Learning Standards for Mathematics and the PARCC Model Content Framework for Geometry. The content standards define what students should understand and be able to do at the high school level; the Model Content Framework describes which content is included and emphasized within the Geometry course, specifically. More information about the relationship between the NYS CCLS and the PARCC Model Content Frameworks can be found in this memo.

For high school mathematics, the standards are organized at three levels: conceptual categories, domains and clusters.

Geometry is associated with high school content standards within the conceptual category of Geometry. This conceptual category contains domains of related clusters of standards. This chart shows the high school mathematics domains included in Geometry, as well as the corresponding percent of credits on the Geometry Regents Exam:

<table>
<thead>
<tr>
<th>Conceptual Category</th>
<th>Domains in Geometry</th>
<th>Percent of Test By Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>Congruence (G-CO)</td>
<td>27%-34%</td>
</tr>
<tr>
<td></td>
<td>Similarity, Right Triangles, and Trigonometry (G-SRT)</td>
<td>29%-37%</td>
</tr>
<tr>
<td></td>
<td>Circles (G-C)</td>
<td>2%-8%</td>
</tr>
<tr>
<td></td>
<td>Expressing Geometric Properties with Equations (G-GPE)</td>
<td>12%-18%</td>
</tr>
<tr>
<td></td>
<td>Geometric Measurement and Dimensions (G-GMD)</td>
<td>2%-8%</td>
</tr>
<tr>
<td></td>
<td>Modeling with Geometry (G-GMD)</td>
<td>8%-15%</td>
</tr>
</tbody>
</table>

The conceptual category of Modeling is also included in Geometry, but is best interpreted not as a collection of isolated topics but rather in relation to other standards:

| Modeling            | Specific modeling domains, clusters, and standards, indicated by a star symbol(★). |

For more information about modeling at the high school level, please consult the High School Progression on Modeling.
Common Core State Standards (Cont.)

Not all of the content in a given course is emphasized equally in the standards. The list of content standards for each course is not a flat, one-dimensional checklist; this is by design. There are sometimes strong differences of emphasis even within a single domain. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material for each course allows depth in learning, which is carried out through the Standards for Mathematical Practice. Without such focus, attention to the practices would be difficult and unrealistic, as would best practices like formative assessment.

The chart below shows:

- Clusters and standards associated with Geometry within each domain. Clusters are identified as Major Content, Supporting Content, or Additional Content, to indicate emphasis within the course.

- NYSED standards clarifications following the applicable standards. More information about the NYSED standards clarifications can be found here.

Note: The Standards for Mathematical Practice form an important part of the Geometry course, as well:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Mathematics – Geometry

Congruence (G-CO)

A. Experiment with the transformations in the plane.

G.CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

NYSED: Trapezoid is defined as “A quadrilateral with at least one pair of parallel sides.”

G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

B. Understand congruence in terms of rigid motions.

G.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Common Core State Standards (Cont.)

Congruence (G-CO) Continued

C. Prove geometric theorems.

G.CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

**NYSED:** Theorems include but are not limited to the listed theorems.
Example: theorems that involve complementary or supplementary angles.

G.CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

**NYSED:** Theorems include but are not limited to the listed theorems.
Example: an exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles of the triangle.

G.CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

**NYSED:** Theorems include but are not limited to the listed theorems.
Example: rhombus is a parallelogram with perpendicular diagonals.

D. Make geometric constructions.

G.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

**NYSED:** Constructions include but are not limited to the listed constructions.
Example: constructing the median of a triangle or constructing an isosceles triangle with given lengths.

G.CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Common Core State Standards (Cont.)

Common Core State Standards for Mathematical Practice

1  Make sense of problems and persevere in solving them. Mathematically proficient students

➢ Explain to themselves the meaning of a problem and look for entry points to its solution.
➢ Explain correspondences between equations, verbal descriptions, tables, and graphs, or draw diagrams of important features and relationships.
➢ Check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”
➢ Understand the approaches of others to solving complex problems and identify correspondences between approaches.

2  Reason abstractly and quantitatively. Mathematically proficient students

➢ Make sense of quantities and their relationships in problem situations. Create a coherent representation of the problem at hand, consider the units involved, attend to the meaning of quantities, and flexibly use different properties of operations and objects.
➢ Decontextualize a given situation and represent it symbolically, and also contextualize to probe into the referents for the symbols involved.

3  Construct viable arguments and critique the reasoning of others. Mathematically proficient students

➢ Understand and use stated assumptions, definitions, and previously established results in constructing arguments.
➢ Compare the effectiveness of two arguments, distinguish correct from incorrect reasoning and, if there is a flaw in an argument, explain what it is.

4  Model with mathematics. Mathematically proficient students

➢ Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
➢ Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
➢ Identify important quantities and their relationships in a practical situation using such tools as diagrams, tables, graphs, flowcharts, and formulas.

5  Use appropriate tools strategically. Mathematically proficient students

➢ Use technology to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
➢ Identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.
➢ Use technological tools to explore and deepen their understanding of concepts.

6  Attend to precision. Mathematically proficient students

➢ Communicate precisely to others. They use clear definitions in discussion with others and in their own reasoning.
➢ State the meaning of the symbols they choose. They are careful about specifying units.
➢ Calculate accurately and efficiently, and express numerical answers with the appropriate degree of precision.

7  Look for and make use of structure. Mathematically proficient students

➢ Look closely to discern a pattern or structure.
➢ Step back for an overview and shift perspective.

8  Look for and express regularity in repeated reasoning. Mathematically proficient students

➢ Notice if calculations are repeated, and look both for general methods and for shortcuts.
➢ When solving problems, maintain oversight of the process while attending to the details, and continually evaluate the reasonableness of results.
# Bloom's Revised Taxonomy: Mathematics

<table>
<thead>
<tr>
<th>New Terms</th>
<th>Actions</th>
<th>Learning Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Creating</strong></td>
<td>Designing, Constructing, Planning, Producing, Inventing, Devising, Making</td>
<td>Creating: (Generating new ideas, products, or ways of viewing things) How could we determine the number of pennies in a jar without counting them? Apply and integrate several different strategies to solve a mathematical problem. Design a new monetary system or an experiment for establishing... Designing, constructing, planning, producing, inventing. Invent a machine to do a specific task. Develop a menu for a new healthy foods restaurant.</td>
</tr>
<tr>
<td><strong>Evaluating</strong></td>
<td>Checking, Hypothesizing, Critiquing, Experimenting, Judging, Testing, Detecting, Monitoring</td>
<td>Evaluating: (Judging the value of a product fora given purpose, using definite criteria) Develop a proof... and justify each step... Using a definition... determine... Justifying a decision or course of action, checking, hypothesizing, critiquing, experimenting, judging. What criteria would you use to evaluate if your answer is correct? Prepare a list of criteria to judge... Evaluate expressions.</td>
</tr>
<tr>
<td><strong>Analyzing</strong></td>
<td>Comparing, Organizing, Deconstructing, Attributes, Outlining, Structuring, Integrating</td>
<td>Analyzing: (Breaking information into parts to explore understandings and relationships) Given a math word problem, determine the strategies that would be necessary to solve it. Write a paragraph describing the relationship... How does... compare to... Comparing, organizing, deconstructing, interrogating, finding. Design a survey to find out... Graph your results. Use a Venn Diagram to show how two topics are the same and different. Translate between visual representations, sentences, and symbolic notation. Make predictions based on experimental or statistical data.</td>
</tr>
<tr>
<td><strong>Applying</strong></td>
<td>Implementing, Carrying out, Using, Executing</td>
<td>Applying: (Using information in concrete situations) Compute the area of actual circles. Use the given graph to... Choose and describe the best method to... Using information in another familiar situation, implementing, carrying out, using, executing. Draw a diagram which shows these fractions or take photographs of the fractions. Determine measures of central tendency and dispersion. Write a journal entry. Write an explanation about this topic for others.</td>
</tr>
<tr>
<td><strong>Understanding</strong></td>
<td>Interpreting, Examining, Summarizing, Inferring, Paraphrasing, Classifying, Comparing, Explaining</td>
<td>Understanding: (Grasping the meaning of material) Given the mathematical formula for the area of a circle, paraphrase it using your own words. Select the graph that illustrates. Explaining ideas or concepts: Interpreting, summarizing, paraphrasing, classifying, explaining. Find items that you can use to show the fractions. Relate or write in your own words... Report to the class... Write a summary report of the event.</td>
</tr>
<tr>
<td><strong>Remembering</strong></td>
<td>Recognizing, Listing, Describing, Identifying, Retrieving, Naming, Locating, Finding</td>
<td>Remembering: (Remembering previously learned material) State the formula for the area of a circle. State the rule of... explain and use the procedure for... Recalling information, recognizing, listing, describing, retrieving, naming, finding, locating. List the fractions you know and can show. List the attributes of your shape. Make a concept map of the topic. Make a chart showing...</td>
</tr>
<tr>
<td>Grade: Mathematics Lesson/Unit Title:</td>
<td>Overall Rating:</td>
<td></td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>----------------</td>
<td></td>
</tr>
<tr>
<td>I. Alignment to the Rigor of the CCSS</td>
<td>I. Alignment to the Rigor of the CCSS</td>
<td>II. Key Areas of Focus in the CCSS</td>
</tr>
<tr>
<td>Observations and Comments:</td>
<td>Observations and Comments:</td>
<td>Suggestions for Improvement:</td>
</tr>
<tr>
<td>The lesson/unit aligns with the letter and spirit of the CCSS:</td>
<td>The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:</td>
<td></td>
</tr>
<tr>
<td>□ Targets a set of grade level mathematics standard(s) at the level of rigor in the CCSS for teaching and learning. **</td>
<td>□ Focus: Centers on the concepts, foundational knowledge, and level of rigor that are prioritized in the standards. **</td>
<td></td>
</tr>
<tr>
<td>□ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed. **</td>
<td>□ Coherence: Makes connections and provides opportunities for students to transfer knowledge and skills within and across domains and learning progressions.</td>
<td></td>
</tr>
<tr>
<td>□ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.</td>
<td>□ Rigor: Requires students to engage with and demonstrate challenging mathematics in the following ways:</td>
<td></td>
</tr>
<tr>
<td>Suggestions for improvement:</td>
<td>□ Application: Provides opportunities for students to independently apply mathematical concepts in real-world situations and problem solving with persistence, choosing and applying an appropriate model or strategy to new situations.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Conceptual Understanding: Requires students to demonstrate conceptual understanding through complex problem solving, in addition to writing and speaking about their understanding.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Procedural Skill and Fluency: Expects, supports, and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.</td>
<td></td>
</tr>
<tr>
<td>Rating: 3 2 1 0</td>
<td>Rating: 3 2 1 0</td>
<td>Rating: 3 2 1 0</td>
</tr>
</tbody>
</table>

Rating Scale for Each Dimension:
1. Meets all "must have" criteria (**) and most of the other criteria in the dimension.
2. Meets many of the "must have" criteria and many of the other criteria in the dimension.
3. Meets some of the criteria in the dimension.
4. Does not meet the criteria in the dimension.

Overall Rating for the Lesson/Unit:
E: Exceptional Lesson/Unit - meets all the "must have" criteria (**) and most of the other criteria in all four dimensions (mainly 3’s and 4’s).
T: Top Lesson/Unit - needs some improvement in one or more dimensions (mainly 2’s and 3’s).
R: Reasonable Lesson/Unit - does not meet the criteria in one to two dimensions (mainly 1’s and 2’s).
N/R: Not ready to review - use subject to revise and organize lesson/unit then resubmit for a quality review.

Quality Rubric developed by Tri-State Collaborative (MA, NY, RI - facilitated by Achieve) 7/8/2012.

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### Tri-State Quality Review Rubric for Lessons & Units: Mathematics – Version 4.2

<table>
<thead>
<tr>
<th>Grade</th>
<th>Mathematics Lesson/Unit Title</th>
<th>Overall Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### III. Instructional Supports

- **Observations and Comments:**
  - The lesson/unit consistently assesses whether students are mastering standards-based content and skills.
  - The lesson/unit regularly assesses whether students are meeting standards-based content and skills.

- **Suggestions for Improvement:**
  - A unit or longer lesson should:
    - Use varied modes of curriculum embedded assessments that may include pre-, formative, summative and self-assessment measures.
    - A unit or longer lesson should:
      - Use varied modes of curriculum embedded assessments that may include pre-, formative, summative and self-assessment measures.

---

#### Rating Scale for Each Dimension:

1. **3:** Meets all "must have" criteria and most of the other criteria in the dimension.
2. **2:** Meets some of the other criteria in the dimension.
3. **1:** Meets most of the "must have" criteria and many of the other criteria in the dimension. (mainly 1s)
4. **0:** Does not meet the criteria in the dimension.

---

**Overall Rating for the Lesson/Unit:**

- **E:** Excellent (Lessons/Unit) - meets all the "must have" criteria (**) and most of the other criteria in all four dimensions (mainly 2s).
- **G/1:** Good (improved) - needs some improvement in one or more dimensions (mainly 1s and 2s).
- **R:** Needs revision - is in "work in progress" and requires significant revision in one or more dimensions (mainly 2s and 1s).
- **N:** Not recommended - does not meet the criteria in the dimensions (mainly 0s and 1s).
- **NR:** Not ready to review - use rubrics to rework and organize lessons/unit then resubmit for a quality review.

---

Quality Rubric developed by Tri-State Collaborative (IA, MN, WI, RI) facilitated by Achieve, 1/21/2012

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Altitudes in a Triangle

SKETCH AND INVESTIGATE

01 If an altitude falls outside the triangle, the triangle is obtuse.

02 When \( \angle A \) is right, altitude \( BD \) lies on side \( BA \).

Note: An altitude tool is provided on the accompanying disk. For tips on making and using custom tools, see the appropriate sections in the help system. (Choose Toolbox from the Help menu, then click on the Custom Tools link.)

03 When the triangle is acute, the three altitudes intersect at the same point inside the triangle.

04 When the triangle is obtuse, two altitudes fall outside the triangle and the third falls inside.

05 All three lines containing the altitudes always intersect at the orthocenter of the triangle. The orthocenter lies inside an acute triangle and outside an obtuse triangle.

EXPLORE MORE

1. For tips on making and using custom tools, see the appropriate sections in the help system. (Choose Toolbox from the Help menu, then click on the Custom Tools link.) This custom tool will be especially useful in the activity The Euler Segment. A sample Orthocenter tool is included in Altitudes.gsp.

2. Each point is the orthocenter of the other three. In other words, if point \( D \) is the orthocenter of \( \triangle ABC \), then point \( C \) is the orthocenter of \( \triangle ABD \), point \( B \) is the orthocenter of \( \triangle ACD \), and point \( A \) is the orthocenter of \( \triangle BCD \).

3. Students should calculate \((1/2)bh\) for each of the three bases and corresponding altitudes.
Altitudes in a Triangle (Cont.)

In this activity you'll discover some properties of altitudes in a triangle. An altitude is a perpendicular segment from a vertex of a triangle to the opposite side (or to a line containing the side). The side where the altitude ends is the base for that altitude, and the length of the altitude is the height of the triangle from that base. Because a triangle has three sides, it also has three altitudes. You'll construct one altitude and make a custom tool for the construction. Then you'll use your tool to construct the other two.

**SKETCH AND INVESTIGATE**

1. Construct triangle $ABC$.
2. Construct a line perpendicular to $AC$ through point $B$.
   
   $Q_1$ As long as your triangle is acute, this perpendicular line should intersect a side of the triangle. Drag point $B$ so that the line falls outside the triangle. Now what kind of triangle is it?
3. With the perpendicular line outside the triangle, use a line to extend side $AC$ so that it intersects the perpendicular line.
4. Construct point $D$, the point of intersection of the extended side and the perpendicular line.
5. Hide the lines.
6. Construct $BD$. Segment $BD$ is an altitude.

7. Drag vertices of the triangle and observe how your altitude behaves.

$Q_2$ Where is your altitude when $\angle A$ is a right angle?
8. Drag your triangle so that it is acute again (with the altitude falling inside the triangle).
Altitudes in a Triangle (Cont.)

9. Make a custom tool for this construction. Name the tool “Altitude.”

10. Use your custom tool on the triangle’s vertices to construct a second altitude.
    Don’t worry if you accidentally construct the altitude that already exists. Just use the tool on the vertices again in a different order until you get another altitude.

11. Use your Altitude tool to construct the third altitude in the triangle.

12. Drag the triangle and observe how the three altitudes behave.

Q3 What do you notice about the three altitudes when the triangle is acute?

Q4 What do you notice about the altitudes when the triangle is obtuse?

When the triangle is obtuse, the three altitudes don’t intersect. But do you think they would if they were long enough? Follow the steps below to investigate that question.

13. Make sure the triangle is obtuse. Construct two lines that each contain an altitude.

14. Construct their point of intersection. This point is called the orthocenter of the triangle.

15. Construct a line containing the third altitude.

16. Drag the triangle and observe the lines.

Q5 What do you notice about the lines containing the altitudes?

EXPLORE MORE

1. Hide everything in your sketch except the triangle and the orthocenter. Make and save a custom tool called “Orthocenter.” You can use this tool in other investigations of triangle centers.

2. In your sketch, draw segments from each vertex to the orthocenter. Now you have three new triangles along with your original triangle. Choose your orthocenter tool from the Custom Tools menu and use it on each of these three new triangles. What do you notice?

3. Do you know a formula for the area of a triangle? Build three different expressions, using different altitudes and bases, that give the area of the triangle.
Constructing Regular Polygons

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Central angle measure</th>
<th>Interior angle measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td>120°</td>
<td>60°</td>
</tr>
<tr>
<td>Square</td>
<td>90°</td>
<td>90°</td>
</tr>
<tr>
<td>Regular pentagon</td>
<td>72°</td>
<td>108°</td>
</tr>
<tr>
<td>Regular hexagon</td>
<td>60°</td>
<td>120°</td>
</tr>
<tr>
<td>Regular octagon</td>
<td>45°</td>
<td>135°</td>
</tr>
<tr>
<td>Regular nonagon</td>
<td>40°</td>
<td>140°</td>
</tr>
<tr>
<td>Regular decagon</td>
<td>36°</td>
<td>144°</td>
</tr>
<tr>
<td>Regular (n)-gon</td>
<td>(360°/n)</td>
<td>((n-2)\times 180°/n)</td>
</tr>
</tbody>
</table>

EXPLORE MORE

1. Students should use Sketchpad’s Calculator to calculate \(5 \times 180°/7\) or \(900°/7\) for an interior angle, or \(360°/7\) for a central angle. They must choose degrees in the Units pop-up menu in the Calculator so that Sketchpad will recognize the calculation as an angle measure. Then they can select this measurement and choose Mark Angle in the Transform menu. Now they can construct the regular heptagon by rotating by this angle.

\(900°/7\) (or \(360°/7\) for the interior angle) is a rational number with an infinite repeating decimal. If students try to rotate by a fixed angle, they can enter only a decimal approximation in the Rotate dialog box. After seven rotations, the error of this approximation will become evident and the resulting polygon won’t be perfectly regular.

2. Euclid’s Proposition 1—Constructing an Equilateral Triangle shows one Euclidean construction. It is possible to construct all the other regular polygons in this table using only the Euclidean tools. The regular pentagon is perhaps the biggest challenge. (It is not possible to construct a regular heptagon using only Euclidean tools.)
Constructing Regular Polygons (Cont.)

A regular polygon is a polygon whose sides all have equal length and whose angles all have equal measure. The easiest way to construct regular polygons with Sketchpad is to use rotations. The figures below show pentagons constructed by two different methods.

![Sketchpad construction of a regular pentagon]

This pentagon was constructed by rotating vertex \( B \) around center point \( A \). \( \angle BAP \) is a central angle of the polygon.

This pentagon was constructed by rotating vertex \( B' \) around vertex \( A \). \( \angle BAP \) is an interior angle of the polygon.

Experiment with using rotations to construct different regular polygons. Figure out central angle measures to rotate by and interior angle measures to rotate by. Each time you make a polygon that seems correct, drag points to make sure it holds together. Make custom tools of your successful constructions to use in later work. Fill in the chart below with the central and interior angle measures for the named regular polygons, whether you have time to construct them all or not. Indicate which constructions you made custom tools for.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Central angle measure</th>
<th>Interior angle measure</th>
<th>Saved tool? (Y or N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular pentagon (5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular hexagon (6)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular octagon (8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular nonagon (9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular decagon (10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular ( n )-gon</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**EXPLORE MORE**

1. The regular heptagon (seven sides) doesn’t appear on the chart because the angle measures aren’t “nice.” What are they? To construct the regular heptagon, use the Calculator to calculate an expression for the desired angle, then mark that measurement as an angle for rotation.
Constructing Regular Polygons (Cont.)

2. Find a book that shows Euclidean constructions for regular polygons and try these constructions using Sketchpad.
Exterior Angles in a Polygon

SKETCH AND INVESTIGATE

01 The sum of the measures of the exterior angles in any convex polygon is $360^\circ$. Make sure students try this investigation with other polygons in addition to the pentagon shown in the example. They could actually manipulate their pentagon into a quadrilateral or triangle, making one or more angle measures disappear.

If students print out their polygons with the exterior angles, they can actually cut out the exterior angles and rearrange them around a single point to show that they sum to $360^\circ$.

02 As you dilate the figure toward any point, the polygon will shrink toward that point, leaving you with only the angles surrounding a common vertex. This has the same visual effect as zooming out from the polygon.

When the polygon appears to be the size of a point, the exterior angles appear as spokes radiating from the point. This provides a visual proof that their sum is $360^\circ$, or the total number of degrees in one revolution around a point.

EXPLORE MORE

1. Interestingly, this same conjecture applies to concave polygons, too, if you consider the “exterior” angles that fall inside the polygon to be negative. To investigate this with Sketchpad, be sure to set Preferences to show angle measures in directed degrees. The figure below demonstrates how selection order determines the sign of an angle measure. If you think in terms of a rotation from one ray to the other, a counter-clockwise rotation is positive and a clockwise rotation is negative.

\[
\begin{align*}
\text{m\_CAB} &= 33.7^\circ \\
\text{m\_BAC} &= -33.7^\circ
\end{align*}
\]
Exterior Angles in a Polygon (Cont.)

An exterior angle of a polygon is formed when one of the sides is extended. Exterior angles lie outside a convex polygon. In this investigation you'll discover the sum of the measures of the exterior angles in a convex polygon.

Do this investigation with a triangle, a quadrilateral, or a pentagon. Plan together with classmates at nearby computers to investigate different polygons so that you can compare your results. The activity here shows a pentagon. Don't let that throw you if you're investigating a triangle or a quadrilateral—the basic steps are the same.

**SKETCH AND INVESTIGATE**

1. Use the **Ray** tool to construct a polygon with each side extended in one direction. Be sure to construct the polygon without creating any extra points. Your initial sketch should have the same number of points (vertices) as sides. If your polygon didn’t end up convex, drag a vertex to make it convex.

   ![Diagram](image.png)

   **m\_\text{BAJ} = 71.4°**
   **m\_\text{CBF} = 82.6°**
   **m\_\text{DCG} = 57.8°**
   **m\_\text{EDH} = 65.5°**
   **m\_\text{AEI} = 82.7°**

2. Construct a point on each ray outside of the polygon so that you’ll be able to measure exterior angles.

3. Measure each exterior angle. Be careful to measure the correct ones!

4. Calculate the sum of the exterior angles.

5. Drag different vertices of your polygon and observe the angle measures and their sum. Be sure the polygon stays convex.

6. Compare your observations with those of classmates who did this investigation with different polygons.
Exterior Angles in a Polygon (Cont.)

Q1 Write a conjecture about the sum of the measures of the exterior angles in any polygon.

Follow the steps below for another way to demonstrate this conjecture.

7. Mark any point in the sketch as a center for dilation.
8. Select everything in the sketch except for the measurements.
9. Change your Arrow tool to the Dilate Arrow tool and use it to drag any part of the construction toward the marked center. Keep dragging until the polygon is nearly reduced to a single point.

Q2 Write a paragraph explaining how this demonstrates the conjecture you made in Q1.

EXPLORE MORE

1. Investigate the sum of the exterior angle measures in concave polygons. For this investigation, you may want to measure angles in directed degrees. The sign of an angle measured in directed degrees depends on the order in which you select points.
Introducing Transformations

SKETCH AND INVESTIGATE

01 The original image and the translated image have the same shape, size, and orientation. They both face the same way. The only difference between them is that they are in different locations. The translated image is a copy of the original after a slide.

02 The original image and the rotated image have the same shape, size, and orientation. The only difference between them is that the rotated image is turned.

03 The original image and the reflected image have the same shape and size, but different orientations. The reflected image is flipped to face in the opposite direction. It is the mirror image of the original.

04 The translated image lies directly on top of the original when the translation vector has length zero. The rotated image lies directly on top of the original when the angle of rotation is zero. The reflected image cannot lie directly on top of the original unless the original shape has reflection symmetry and the mirror line is a line of symmetry.

EXPLORE MORE

1. Answers will vary. Ask students to describe the different transformations in their designs. (Perhaps you might require that each design contain at least one example of each of the three basic isometries.)

2. Two reflections in succession over intersecting lines are equivalent to a single rotation. See the activity Reflections over Two Intersecting Lines.

3. Two reflections in succession over parallel lines are equivalent to a single translation. See the activity Reflections over Two Parallel Lines.
Introducing Transformations (Cont.)

A transformation is a way of moving or changing a figure. There are three types of basic transformations that preserve the size and shape of the figure. These three, reflections, rotations, and translations, are called isometries. Isometries flip, turn, or slide a figure but never bend or distort it. In this activity you'll experiment with basic isometries by transforming a flag-shaped polygon. (You'll use this shape because it's easy to keep track of where it's pointing.)

SKETCH AND INVESTIGATE: TRANSLATIONS

1. Construct the vertices of a flag shape and construct its interior.

2. In order to translate a shape, you need to indicate a direction and a distance. To do this, construct segment $AB$. Then select, in order, point $A$ and point $B$. In the Transform menu, choose Mark Vector.

3. Select the interior of the flag; then, in the Transform menu, choose Translate. Make sure By Marked Vector is checked in the Translate dialog box, then click Translate.

4. Change the color of the translated image.

5. Label the translated polygon Translated image.

6. Drag point $B$ to change your vector, and observe the relationship between the translated image and the original figure.

Q1 Compare the translated image to the original figure. How are they different and how are they the same?
Introducing Transformations (Cont.)

SKETCH AND INVESTIGATE: ROTATIONS

7. In order to rotate a shape, you need to indicate a center of rotation and an angle of rotation. Start by creating angle $ECD$ using two attached segments, as indicated below.

8. Mark point $C$ as a center of rotation.

9. Mark $\angle ECD$ as an angle of rotation.

10. Rotate the original flag-shaped interior by the marked angle.

11. Change the color of the rotated image and label it Rotated image.

12. Drag point $D$ to change your angle, and observe the relationship between the rotated image and the original figure.

Q2 Compare the rotated image to the original figure. How are they different and how are they the same?

SKETCH AND INVESTIGATE: REFLECTIONS

13. To reflect a shape, you need a mirror line (also called a line of reflection). Draw a line and label it Mirror line.

14. Mark the line as a mirror.

15. Reflect the original flag-shaped interior. Your image may end up off the screen. If so, move the original figure closer to the mirror line.

16. Change the color of the reflected image. Label it Reflected image.
Introducing Transformations (Cont.)

17. Drag your mirror line, and observe the relationship between the reflected image and the original figure.

03 Compare the reflected image to the original figure. How are they different and how are they the same?

04 Explain whether it is possible for any of the three images in your sketch to lie directly on top of one another. Experiment by dragging different parts of your sketch.

EXPLORE MORE

1. Use reflections, rotations, translations, or combinations of these transformations to make a design.

2. Reflect a figure over a line, then reflect the image over a second line that intersects the first. What single transformation would take your original figure to the second reflected image?

3. Reflect a figure over a line, then reflect the image over a second line that is parallel to the first. What single transformation is the same as this combination of two reflections?
Proportions with an Angle Bisector in a Triangle

SKETCH AND INVESTIGATE

01 No, point D is not the midpoint of BC.

02 Point D is the midpoint of BC when triangle ABC is isosceles with AB = AC.

03 When AB is greater than AC, BD is greater than CD.

04 \( \frac{AB}{BD} = \frac{BD}{CD} \)

05 An angle bisector in a triangle divides the opposite side in the same ratio as the ratio of the lengths of the sides of the bisected angle.

EXPLORE MORE

1. Construct a triangle with the given segment as one side and the other two sides in a ratio of 2:3. Bisect the angle formed by those two sides.
Proportions with an Angle Bisector in a Triangle (Cont.)

Quick! Where does an angle bisector intersect the opposite side in a triangle? Did you guess the midpoint? A quick check will show that this works only in special cases. In this activity, you'll discover a proportion relationship of angle bisectors in triangles.

**SKETCH AND INVESTIGATE**

1. Construct $\triangle ABC$.
2. Construct the bisector of $\angle BAC$.
3. Construct point $D$ where the bisector intersects $\overline{BC}$.
4. Drag vertices of the triangle and observe point $D$.

   Q1 Is point $D$ the midpoint of $\overline{BC}$?

5. Hide $\overline{BC}$ and construct $\overline{BD}$ and $\overline{DC}$.
6. Measure $AB$, $BD$, $DC$, and $AC$.

   Q2 Drag some more. Under what conditions will point $D$ be the midpoint of $\overline{BC}$?

   Q3 How do $BD$ and $CD$ compare when $AB$ is greater than $AC$?

   Q4 Your answer to Q3 may give you an idea for a proportion. Measure ratios involving $AB$, $AC$, $BD$, and $CD$ to see if you can create equal ratios (a proportion).

7. Manipulate your triangle to make sure your proportion holds for any triangle.

   Q5 Write your findings as a conjecture.

**EXPLORE MORE**

1. Construct a segment. Use your conjecture to figure out a construction that divides the segment into a given ratio, say 2:3.
Triangle Congruence

SKETCH AND INVESTIGATE

01 No. Three sides determine a unique triangle.

02 If two triangles have three pairs of congruent sides, then the triangles are congruent (SSS).

03 These combinations of parts guarantee congruence: SSS, SAS, ASA, AAS. For AAS, it's important to state that the sides must correspond. (The correspondence is forced by the order in the cases: It's impossible for the parts not to correspond.)

SSA and AAA do not guarantee congruence.

EXPLORE MORE

1. Counterexample for SSA:

   ![Counterexample for SSA]

   Counterexample for AAA:

   ![Counterexample for AAA]
Triangle Congruence (Cont.)

If the three sides of one triangle are congruent to three sides of another triangle (SSS), must the two triangles be congruent? What if two sides and the angle between them in one triangle are congruent to two sides and the angle between them in another triangle (SAS)? Which combinations of parts guarantee congruence and which don't? In this activity you'll investigate that question.

**SKETCH AND INVESTIGATE**

1. Open the sketch Triangle Congruence.gsp. You'll see a figure like the one shown below, along with some text.

   \[ \text{Side-Side-Side} \]

   ![Side-Side-Side Diagram]

2. Read the text in the sketch and follow the instructions to try to make a triangle DEF that is not congruent to \( \triangle ABC \).

   **Q1** Could you form a triangle with a different size or shape given the three sides? __________

   **Q2** If three sides of one triangle are congruent to three sides of another (SSS), what can you say about the triangles? Write a conjecture that summarizes your findings.

3. Go to the SAS page and make a triangle DEF with those given parts. Try to make a triangle that is not congruent to \( \triangle ABC \).

4. Experiment on each of the other pages.

   **Q3** Of SSS, SAS, SSA, ASA, AAS, and AAA, which combinations of corresponding parts guarantee congruence in a pair of triangles? Which do not?

**EXPLORE MORE**

1. For each of the combinations of parts that do not necessarily guarantee congruence, sketch a pair of noncongruent triangles with those congruent parts. Explain why the triangles are not congruent.
Triangle Inequalities

**SKETCH AND INVESTIGATE**

01 If the sum of two side lengths equals the third side length, two sides of the triangle collapse onto the third side, forming a segment instead of a triangle.

02 It’s impossible for the sum of the lengths of any two sides of a triangle to be less than the length of the third side. If the sum of the lengths of the two segments (sides) is less than the length of the third side, the two short segments together are not long enough to meet and create a closed shape.

03 The length of any one side of a triangle must be less than the sum of the lengths of the other two sides.

04 Students should fill in the chart with the following angles.
   First row: $\angle C$, $\angle B$, $\angle A$; Second row (the same): $\angle C$, $\angle B$, $\angle A$.

05 The largest angle in a triangle is always opposite the longest side and the smallest angle is always opposite the shortest side.

**EXPLORE MORE**

1. The longest median and the longest altitude are to the shortest side. The shortest median and the shortest altitude are to the longest side.
Triangle Inequalities (Cont.)

In this investigation you'll discover relationships among the measures of the sides and angles in a triangle.

SKETCH AND INVESTIGATE

1. Construct a triangle.
   \[ m_{AB} = 2.7 \text{ cm} \]
   \[ m_{BC} = 3.6 \text{ cm} \]
   \[ m_{CA} = 3.4 \text{ cm} \]

2. Measure the lengths of the three sides.

3. Calculate the sum of any two side lengths.
   \[ m_{AB} + m_{BC} = 6.31 \text{ cm} \]

4. Drag a vertex of the triangle to try to make the sum you calculated equal to the length of the third side.

Q1 Is it possible for the sum of two side lengths in a triangle to be equal to the third side length? Explain.

Q2 Do you think it's possible for the sum of the lengths of any two sides of a triangle to be less than the length of the third side? Explain.

Q3 Summarize your findings as a conjecture about the sum of the lengths of any two sides of a triangle.

5. Measure \( \angle ABC, \angle BAC, \) and \( \angle ACB \).

6. Drag the vertices of your triangle and look for relationships between side lengths and angle measures.

Q4 In each area of the chart below, a longest or shortest side is given. Fill in the chart with the name of the angle with the greatest or least measure, given that longest or shortest side.

<table>
<thead>
<tr>
<th>( AB ) longest side</th>
<th>Largest angle?</th>
<th>( AC ) longest side</th>
<th>Largest angle?</th>
<th>( BC ) longest side</th>
<th>Largest angle?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( AB ) shortest side</td>
<td>Smallest angle?</td>
<td>( AC ) shortest side</td>
<td>Smallest angle?</td>
<td>( BC ) shortest side</td>
<td>Smallest angle?</td>
</tr>
</tbody>
</table>

Q5 Summarize your findings from the chart as a conjecture.

EXPLORE MORE

1. Investigate inequalities among medians or altitudes in a triangle. Report your findings.